Differential Equations

- An equation involving derivatives of the dependent variable with respect to independent variable (variables) is known as a differential equation.

- Order of a differential equation is the order of the highest order derivative occurring in the differential equation.

- Degree of a differential equation is defined if it is a polynomial equation in its derivatives.

- Degree (when defined) of a differential equation is the highest power (positive integer only) of the highest order derivative in it.

- A function which satisfies the given differential equation is called its solution.
- The solution which contains as many arbitrary constants as the order of the differential equation is called a general solution and the solution free from arbitrary constants is called particular solution.
- To form a differential equation from a given function we differentiate the function successively as many times as the number of arbitrary constants in the given function and then eliminate the arbitrary constants.

- Variable separable method is used to solve such an equation in which variables can be separated completely i.e. terms containing y should remain with dy and terms containing x should remain with dx.

- A differential equation which can be expressed in the form dy/dx = f(x,y) where f(x,y) is a homogenous function of degree zero is called a homogeneous differential equation.

- A differential equation of the form (dy/dx)+Py = Q, where P and Q are constants or functions of x only is called a first order linear differential equation.

- Form the differential equation representing the family of curves $y = a \sin (x + b)$, where a, b are arbitrary constants.

  Solution:-

  We have $y = a \sin (x + b)$

  $dy/dx = a \cos (x + b)$…………………………..(1)

  $d^2y/dx^2 = -\, a \sin (x + b)$………………………(2)

  Eliminating a and b using equations 1 and 2 we get $(d^2y/dx^2) + y = 0$.

- Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$

  Solution:-

  $\frac{dy}{1 + y^2} = \frac{dx}{1 + x^2}$

  Integrating both sides,

  $\int \frac{dy}{1 + y^2} = \int \frac{dx}{1 + x^2}$

  $\tan^{-1}y = \tan^{-1}x + C$

- Find the general solution of the differential equation y dx − (x + 2y^2) dy = 0.

  Solution:-

  (dx/dy) − (x/y) = 2y

  This is of the type (dx/dy) + Px = Q where P = -(1/y) Q = 2y

  Hence IF = e$^{\int(-dy/y)}$ = e$^{-\log y}$ = (1/y)

  The solution of differential equation is

  x(1/y) = ∫(2y)(1/y)dy + C

  (x/y) = ∫2dy + C

  (x/y) = 2y + C

  x = 2y^2 + Cy